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АЛГОРИТМЫ ПРЯМОГО И ОБРАТНОГО ДКП МАЛЫХ ПОРЯДКОВ С УМЕНЬШЕННОЙ МУЛЬТИПЛИКАТИВНОЙ СЛОЖНОСТЬЮ

А. ЦАРЕВ, М. МАКОВСКА, П. СТШЕЛЕЦ

Западно-поморский технологический университет,
Польша, Щецин, 210-71, ул. Жолнерска, д. 52

Аннотация. Дискретные ортогональные преобразования, такие как дискретное преобразование Фурье, дискретное преобразование Уолша, дискретное преобразование Хартли, пилоподобное преобразование, дискретное косинус-преобразование и т. д., являются важными инструментами численного анализа, обработки сигналов и статистических методов. Успешное использование этих преобразований объясняется наличием быстрых алгоритмов для их реализации. Особое место в арсенале дискретных ортогональных преобразований занимают прямое и обратное дискретное косинус-преобразование (ДКП). В статье предлагается ряд параллельных алгоритмов прямого и обратного ДКП. Их синтез основан на удачной факторизации матриц преобразования. Представлено несколько полностью параллельных алгоритмов реализации прямого и обратного ДКП малых порядков для $N = \{2, 3, 4, 5, 6, 7\}$.

Ключевые слова: дискретное косинус-преобразование; ДКП; СБИС-ориентированный алгоритм; быстрые вычисления

1. ВВЕДЕНИЕ

Традиционные виды дискретных ортогональных преобразований такие, как дискретное преобразование Фурье, дискретное преобразование Хартли, дискретные преобразования Уолша и Хаара, пилоподобное (Slant) преобразование, дискретное косинус-преобразование (ДКП) являются важными инструментами обработки данных [1]. В целях минимизации вычислительной сложности и имплементационных ресурсов реализации этих преобразований разработаны различные быстрые алгоритмы [2–4]. Среди арсенала вышеупомянутых преобразований ДКП является одним из наиболее важных [5–9].

ДКП нашло широкое применение во многих научных и технологических областях, включающих компрессию данных [10, 11], цифровую обработку сигналов (ЦОС) и изображений (ЦОИ) [12–14], цифровое телевидение [15–18], цифровые водяные знаки [19–23], телекоммуникацию [24, 25] и др. Быстрые алгоритмы для этого типа преобразований описаны в [1–9, 26–54].

Большинство этих публикаций посвящено алгоритмам ДКП для последовательностей длины 8, поскольку преобразования блоков именно такой длины используются в стандартах компрессии данных [26–31]. Одна часть работ посвящена одномерным и двумерным алгоритмам ДКП, ориентированным на обработку последовательностей данных, длина которых является степенью двойки [32–52], другая часть публикаций касается так называемых «алгоритмов простых множителей» [51–54].

Трудоемкость и время вычисления ДКП явилось причиной появления целого ряда публикаций, посвященных разработке аппаратных ускорителей этого преобразования, реали-

DOI: 10.20535/S0021347019110025

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Received June 26, 2019 Revised November 6, 2019 Accepted November 8, 2019